General Argument concerning Bell's Thought Experiment

21-6-83

Consider two-particle state

$$\Psi_I = \sum_n c_n |a_n\rangle |z_n\rangle \qquad (1)$$

where a_n is eigenvalue of A
z_n . . . B

So Ψ contains value a_n of A on particle 1 with value z_n of B . . . 2.

Now consider $\text{Prob}^{\Psi}_{C,D}(c_i, d_j)$

$$< \text{Prob}^{\Psi}_{C\otimes I}(c_i)$$

$$\left(= \sum_{d_j} \text{Prob}^{\Psi}_{C,D}(c_i, d_j) \right)$$

Now suppose $\text{Prob}^{\Psi}_{C\otimes I}(c_i) = 0$

Then $\text{Prob}^{\Psi}_{C,D}(c_i, d_j) = 0$ for all d_j.

In other words, if we select a subensemble of systems all of which have $I\otimes 1) = d_j$, this will never produce a non-vanishing probability for an associated value c_i of $C\otimes I$ & hence the requirement of $I+D$, the probability of finding the value c_i of $C\otimes I$ near zero.

We now illustrate the general result for the decay problem.

Take $A = B = C =$ p_y (Transverse momentum)

and $D =$ q_y (Transverse position)

We want to show that an exact measurement of q_y on particle 2 never produces more dispersion in p_y on particle ① than is already contained in Ψ.

We consider two examples

① Ψ is a Gaussian

$$\Psi = \int_{-\infty}^{\infty} |p_y\rangle |-p_y\rangle \, dp_y$$

The range of p_y on particle ① is $-\infty$ to $+\infty$.

Since $|p_y\rangle = \frac{1}{\sqrt{2\pi}} \int e^{i q_y p_y} |q_y\rangle \, dq_y$.

Now $\Psi = \frac{1}{2\pi} \iiint dq_y \, dq_y' \, dp_y \, e^{i(q_y - q_y')p_y} |q_y\rangle |q_y'\rangle$

$= \iint dq_y \, dq_y' \, \delta(q_y - q_y') |q_y\rangle |q_y\rangle$

$= \int |q_y\rangle |q_y\rangle \, dq_y$

So measuring q_y exactly on ② makes q_y exact on ① which means dispersion from $-\infty$ to ∞ for p_y on ①.

But this is already present in Ψ
So measuring q_y on (2) does not increase the dispersion in p_y on (1).

(2) Mod realistically

Take $\Psi = \int_{-\infty}^{\infty} |p_y\rangle\langle p_y| e^{-p_y^2/2\sigma} dp_y$.

So p_y integration is 'cut off' at $p_y \sim \sqrt{\sigma}$

Then $c(q_y, q_y') = \langle q_y | \langle q_y' | \Psi \rangle$

$$= \int_{-\infty}^{\infty} e^{i(q_y - q_y')p_y} e^{-p_y^2/2\sigma} dp_y.$$

$$= e^{-\sigma(q_y - q_y')^2/2}$$

this shows that $\text{Prob}^{\Psi}_{q_{y_1}, q_{y_2}}(q_y, q_y')$

$$= |c(q_y, q_y')|^2 = e^{-\sigma(q_y - q_y')^2}.$$

This shows that q_{y_1} and q_{y_2}, the transverse locations of particles (1) and (2) are correlated to an extent $1/\sqrt{\sigma}$.

i.e. Measuring q_{y_2} precisely determines q_{y_1} to within $1/\sqrt{\sigma}$ and this gives an uncertainty in p_{y_1} of $\sim 1/1/\sqrt{\sigma} = \sqrt{\sigma}$

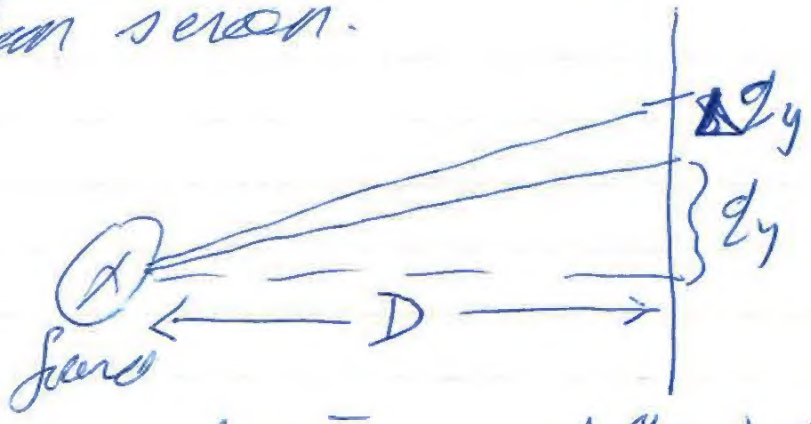
But this again is just the uncertainty in p_{y_1} as specified in the angle $\overline{\gamma}$.

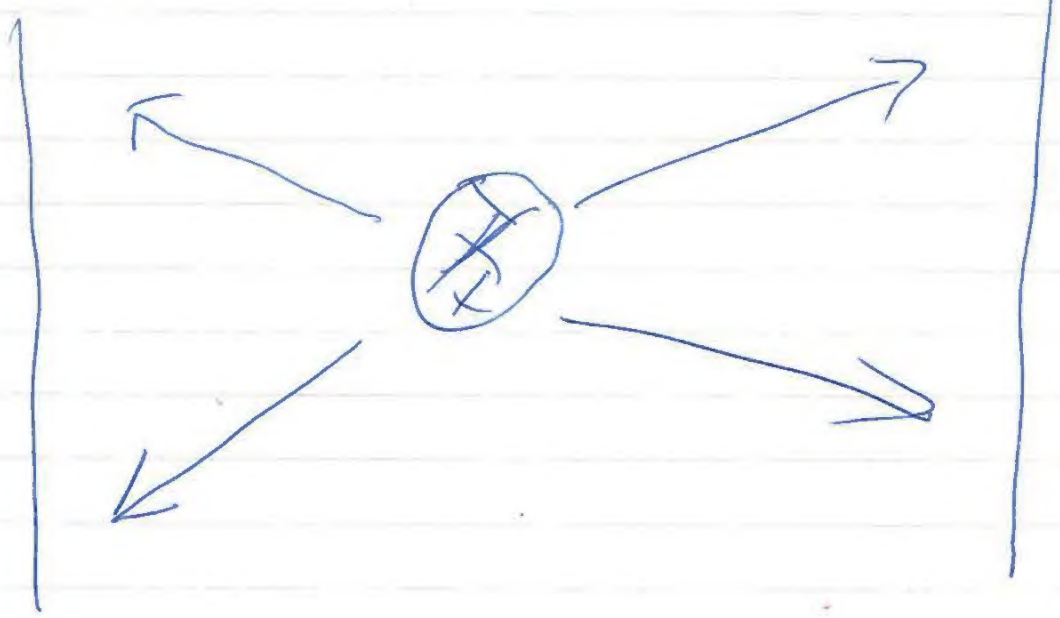
Important note

In a decay experiment we correlate p_{y_1} and p_{y_2} by conservation of momentum but we do not correlate y_1 and y_2 but only ranges. Δy_2 is also associated with Δy_1, where in the above example $\Delta y_1 = \Delta y_2 = 1/\sqrt{5}$

Here is no uncertainty here zero correlation ~~p_{y_1} and p_{y_2} means~~ momenta in a decay experiment mean correlated directions of the outgoing beams and in the opposite region

$\frac{\Delta y}{D} \to 0$ for fixed Δy as $D \to \infty$ where D is distance of source from screen.



Uncertainty in direction $= \frac{\Delta y}{D} \to 0$ as $D \to \infty$

Typically in a decay experiment the source is an extended interaction region so that we should not expect perfect ~~by~~ correlation only by conservation due to conservation of momentum